frequencyConnectedness
Computing Diebold & Yilmaz and Barunik & Krehlik measures of connectedness

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Abstract

To estimate, we build on the vars and urca packages that provide the coefficients estimate of the system. The dependency within the system is then computed based on the forecast error variance decomposition (FEVD) or the generalized FEVD estimates.

We provide parallelized version of the estimation functions that allow for rolling window estimation of the dependency.

Spillovers
Consider a covariance stationary N-variable VAR(p),

\[ x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \epsilon_t \]

with error term having the covariance matrix \( \Sigma \). The system can be represented in MA infinite form as the following,

\[ x_t = \sum_{i=0}^{\infty} A_i \epsilon_{t-i} \]

The KPSS H-step-ahead forecast error variance (Pesaran and Shin, 1998) decomposition is then computed as

\[ \theta_{1,j}(H) = \frac{\sigma_{1,j}^2 \sum_{h=0}^{H} \sum_{i=0}^{\infty} (A_i^\prime \Sigma A_i)^2}{\sigma_{1,j}^2} \]

and the resulting spillover is defined as

\[ S(H) = \frac{100}{H} \sum_{j=1}^{H} \theta_{1,j}(H) \]

Frequency decomposition
Instead of IRFs, we consider a frequency response function, or transfer function we define the generalised causation spectrum over frequencies \( \omega \in [-\pi, \pi) \) as

\[ \frac{\sigma_{1,j}^2 \left( \left( \Psi(e^{-i\omega}) \right)_{i,j} \right)^2}{\left( \left( \Psi(e^{-i\omega}) \Sigma \Psi(e^{i\omega}) \right)_{i,j} \right)^2} \]

Let us also define the spectral power at a given frequency

\[ \Gamma_j(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left( \Psi(e^{-i\omega}) \right)_{j,j} d\omega \]

Now suppose that we have system where it is wide-sense stationary. We establish the correspondence between the original FEVD measure and the decomposed measure as

\[ \theta_{d,j,k} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Gamma_j(\omega) (f(\omega))_{j,k} d\omega \]

The FEVD measure can be arbitrarily decomposed on various frequency bands of interest.

Based on the decomposed FEVD we define two decomposed connectedness measures

\[ C_d^W = 1 - \frac{\sum_{j,k} \theta_{d,j,k}}{\sum_{j,k} \theta_{1,j,k}} \]

Empirical example
We select the most liquid stocks from seven sectors in the American stock market. The selected stocks are: Bank of America Corp, Coca Cola, Microsoft, Pfizer, Walt Disney Co, Exxon Mobile Co, AT&T Inc. The data cover the period 2006 -- 2015, here we cover the turbulent times of crisis as well as the pre- and post-crisis periods.

To capture the heterogeneity of the market transactions we investigate the periodicities up to one week, week to one month, month to one quarter, and longer.

In many cases, the structure of connectedness is dynamically evolving through the time, hence not strictly stationary. To capture this dynamism, we adopt the Starica & Granger (2015) framework of thinking about the time-series and estimate the connectedness within rolling window. We opt for window of length 250 days.

Further work
Currently, the package supports standard computation of the dependence measures. We aim to further provide estimation of the confidence intervals, where we however have to resort to C implementation because of computational exigencies. We will also work to provide facilities to compute directional connectedness as defined in Diebold & Yilmaz (2012) and convenient plotting functions.

Literature

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